TRIGONOMETRY

Trigonometry is made of three words "tri", "gono", "metry", where "tri" means "three", "gono" means "side" and "metry" means measurement. So, trigonometry is study of measuring three side figures which is TRIANGLE.
Usually we use right angle triangle to solve problem based on trigonometry. Problem in trigonometry are usually based on trigonometric ratio.

What is Trigonometric Ratio?

Trigonometric ratio are the ratio between two sides of a triangle. At particular angle the ratio between two sides will remain same irrespective to their length.
There are six Trigonometric Ratios which are as:

Sine: It is a ratio between a perpendicular and hypotenuse. It is represented as "sin" in all trigonometric identities.

$$\sin\theta = \frac{perpendicular}{hypotenuse}$$

Where represents the angle for which the ratio is derived.

Cosine: It is a ratio between a base and hypotenuse. It is represented as "cos" in all trigonometric identities.

$$\cos\theta = \frac{base}{hypotenuse}$$

Secant: It is a ratio between a hypotenuse and base. It is represented as "sec" in all trigonometric identities.

$$\sec\theta = \frac{hypotenuse}{base}$$

Cosecant: It is a ratio between a hypotenuse and perpendicular. It is represented as "cosec" in all trigonometric identities.

$$\text{cosec}\,\theta = \frac{hypotenuse}{perpendicular}$$

Tangent: It is a ratio between a perpendicular and base. It is represented as "tan" in all trigonometric identities.

$$\tan\theta = \frac{perpendicular}{base}$$

Cotangent: It is a ratio between a base and perpendicular. It is represented as "cot" in all trigonometric identities.

cotθ = $\frac{base}{perpendicular}$

Angle:

When two rays (initial and terminal) meet at a point after rotation
in a plane then they are said to have described an angle. In other words we can say, the circular distance between two inclined lines is called angle.

Unit of Angle:

- ⇒ Degree (o)
- ⇒ Radian (c)

Relationship between degree and radian:

π radian = 180°

For below particular angles the value of trigonometric ratios is constant.

Function	0°	30°	45°	60°	90°
Sinθ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tanθ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
cotθ	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
Secθ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
cosecθ	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Signs of Trigonometric Ratio in quadrants:

1st quadrant: All positive
2nd quadrant: sin and cosec positive
3rd quadrant: tan and cot positive
4th quadrant: cos and sec positive

Relation between Trigonometric Ratios:

sinθ × cosecθ = 1

cosθ × secθ = 1

tanθ × cotθ = 1

tanθ = $\frac{sin\theta}{cos\theta} = \frac{sec\theta}{cosec\theta}$

cotθ = $\frac{cos\theta}{sin\theta} = \frac{cosec\theta}{sec\theta}$

Trigonometric Ratios of Allied Angles:

With(θ)

$\sin(-\theta) = -\sin\theta$
$\cos(-\theta) = \cos\theta$
$\tan(-\theta)) = -\tan\theta$
$\cot(-\theta) = -\cot\theta$
$\sec(-\theta) = \sec\theta$
$\text{cosec}(-\theta) = -\text{cosec}\theta$

With ($90°-\theta$)

$\sin(90°-\theta) = \cos\theta$
$\cos(90°-\theta) = \sin\theta$
$\tan(90°-\theta) = \cot\theta$
$\cot(90°-\theta) = \tan\theta$
$\sec(90°-\theta) = \text{cosec}\theta$
$\text{cosec}(90°-\theta) = \sec\theta$

With ($90° + \theta$)

$\sin(90° + \theta) = \cos\theta$
$\cos(90° + \theta) = -\sin\theta$
$\tan(90° + \theta) = -\cot\theta$
$\cot(90° + \theta) = -\tan\theta$
$\sec(90° + \theta) = -\text{cosec}\theta$
$\text{cosec}(90° + \theta) = \sec\theta$

With ($180°-\theta$)

$\sin(180°-\theta) = \sin\theta$
$\cos(180°-\theta) = -\cos\theta$
$\tan(180°-\theta) = -\tan\theta$
$\cot(180°-\theta) = -\cot\theta$
$\sec(180°-\theta) = -\sec\theta$
$\text{cosec}(180°-\theta) = \text{cosec}\theta$

With ($180° + \theta$)

$\sin(180° + \theta) = -\sin\theta$
$\cos(180° + \theta) = -\cos\theta$
$\tan(180° + \theta) = \tan\theta$
$\cot(180° + \theta) = \cot\theta$
$\sec(180° + \theta) = -\sec\theta$
$\text{cosec}(180° + \theta) = -\text{cosec}\theta$

With ($270°-\theta$)

$\sin(270°-\theta) = -\cos\theta$
$\cos(270°-\theta) = -\sin\theta$
$\tan(270°-\theta) = \cot\theta$
$\cot(270°-\theta) = \tan\theta$
$\sec(270°-\theta) = -\text{cosec}\theta$
$\text{cosec}(270°-\theta) = -\sec\theta$

With ($270° + \theta$)

$\sin(270° + \theta) = -\cos\theta$
$\cos(270° + \theta) = \sin\theta$
$\tan(270° + \theta) = -\cot\theta$
$\cot(270° + \theta) = -\tan\theta$
$\sec(270° + \theta) = \text{cosec}\theta$
$\text{cosec}(270° + \theta) = -\sec\theta$

With ($360°-\theta$)

$\sin(360°-\theta) = -\sin\theta$
$\cos(360°-\theta) = \cos\theta$
$\tan(360°-\theta) = -\tan\theta$
$\cot(360°-\theta) = -\cot\theta$
$\sec(360°-\theta) = \sec\theta$
$\text{cosec}(360°-\theta) = -\text{cosec}\theta$

With ($360°+\theta$)

$\sin(360°+\theta) = \sin\theta$
$\cos(360°+\theta) = \cos\theta$
$\tan(360°+\theta) = \tan\theta$
$\cot(360°+\theta) = \cot\theta$
$\sec(360°+\theta) = \sec\theta$
$\text{cosec}(360°+\theta) = \text{cosec}\theta$

Some Useful Identities

1) $sin^2\theta + cos^2\theta = 1$
It can also be expressed as
$sin^2\theta = 1 - cos^2\theta$
$cos^2\theta = 1 - sin^2\theta$

2) $sec^2\theta - tan^2\theta = 1$
It can also be expressed as
$sec^2\theta = 1 + tan^2\theta$
$sec^2\theta - 1 = tan^2\theta$

3) $cosec^2\theta - cot^2\theta = 1$
It can also be expressed as
$cosec^2\theta = 1 + cot^2\theta$
$cosec^2\theta - 1 = cot^2\theta$

4) sin (A + B) = sin A cos B + cos A sin B
5) sin (A – B) = sin A cos B – cos A sin B
6) cos (A + B) = cos A cos B – sin A sin B
7) cos (A – B) = cos A cos B + sin A sin B
8) 2 sin A cos B = sin (A + B) + sin (A – B)
9) 2 cos A sin B = sin (A + B) – sin (A – B)
10) 2 sin A sin B = cos (A – B) –sin (A + B)
11) 2 cos A cos B = cos (A + B) + cos (A -B)
12) sin2 A – sin2 B = sin (A + B) sin (A – B)
13) cos2 A – cos2 B = cos (A+B) cos (A – B)

14) tan (A + B) = $\frac{tanA+tanB}{1-tanAtanB}$

15) tan (A – B) = $\frac{tanA-tanB}{1+tanAtanB}$

16) cot (A + B) = $\frac{cotAcotB-1}{cotA+cotB}$

17) cot (A – B) = $\frac{cotAcotB+1}{cotA-cotB}$

18) sin 2A = 2 sin A cos A = $\frac{2tanA}{1+tan^2A}$

19) cos 2A = cos^2A -sin^2A= $2cos^2A$ –1 = 1 –sin^2A = $\frac{1-tan^2A}{1+tan^2A}$

20) tan 2A = $\frac{2tanA}{1-\ tan^2A}$

21) sin 3A = 3 sin A – 4 sin^3A

22) cos 3A = 4 cos^3A– 3 cos A

23) tan 3A = $\frac{3tanA-tan^3A}{1-3tan^2A}$

24) sin C + sin D = 2 sin($\frac{C+D}{2}$) . cos($\frac{C-D}{2}$)

25) sin C – sin D = 2 cos($\frac{C+D}{2}$) . sin($\frac{C-D}{2}$)

26) cos C + cos D = 2 cos($\frac{C+D}{2}$) . cos($\frac{C-D}{2}$)

27) cos C – cos D = 2 sin($\frac{C+D}{2}$) sin($\frac{C-D}{2}$)

28) If $4\theta < 60$

i. $\sin\theta.\sin2\theta.\sin 4\theta = \frac{1}{4}\sin3\theta$

ii. $\cos\theta.\cos2\theta.\cos4\theta = \frac{1}{4}\cos 3\theta$

iii. $\tan\theta.\tan2\theta.\tan4\theta = \tan3\theta$

iv. $\cot\theta.\cot2\theta.\cot4\theta = \cot3\theta$

29) For all value of θ

i. $\sin(60° - \theta)\sin\theta \cdot \sin(60° + \theta) = \frac{1}{4}\sin 3\theta$

ii. $\cos(60° - \theta)\cos\theta \cdot \cos(60° + \theta) = \frac{1}{4}\cos 3\theta$

iii. $\tan(60° - \theta)\tan\theta \cdot \tan(60° + \theta) = \tan 3\theta$

iv. $\cot(60° - \theta)\cot\theta \cdot \cot(60° + \theta) = \cot 3\theta$

30) If $A + B = 45°$

i. $(1 + \tan A)(1 + \tan B) = 2$

ii. $(1 - \cot A)(1 - \cot B) = 2$

31) If $A + B = 90°$

i. $\sin A = \cos B$

ii. $\operatorname{cosec} A = \sec B$

iii. $\tan A = \cot B$

32) If $A + B + C = 90°$

i. $\tan A.\tan B + \tan B.\tan C + \tan C.\tan A = 1$

ii. $\cot A + \cot B + \cot C = \cot A.\cot B.\cot C$

33) If $A + B + C = 180°$

i. $\tan A + \tan B + \tan C = \tan A.\tan B.\tan C$

ii. $\cot A.\cot B + \cot B.\cot C + \cot C.\cot A = 1$

iii. $\sin 2A + \sin 2B + \sin 2C = 4\sin A.\sin B.\sin C$

34) $\tan(45° + \theta) = \frac{1+\tan\theta}{1-\tan\theta} = \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$

35) $\tan(45° - \theta) = \frac{1-\tan\theta}{1+\tan\theta} = \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$