Average → Basic Concepts and Formulas

As we know that average is a central value of a set of data and the basic formula to calculate the average is sum of the observation is divided by the number of terms. We will now learn about some typical concepts, formulas and shortcuts that can come in handy while solving questions.

Concept 1

Let us recall some formula of arithmetic progression.

- Sum of first n natural numbers = $\frac{n(n+1)}{2}$
- Sum of first n even natural numbers = $n(n+1)$
- Sum of first n odd natural numbers = n^2
- Sum of squares of first n natural numbers = $\frac{n(n+1)(2n+1)}{6}$
- Sum of cubes of first n natural numbers = $\left[\frac{n(n+1)}{2}\right]^2$
- Sum of squares of first n even natural numbers = $\frac{2n(n+1)(2n+1)}{3}$
- Sum of squares of first n odd natural numbers = $\frac{n(2n-1)(2n+1)}{3}$
- Sum of cubes of 1st 'n' Odd natural number = $1^3 + 3^3 + 5^3 + \ldots + (2n-1)^3 = n^2(2n^2-1)$
- Sum of cubes of 1st 'n' Even natural number = $2^3 + 4^3 + 6^3 + \ldots + (2n)^3 = 2[n(n+1)]^2$

(Note: These formulas have wide applications in the Quant and DILR section of all competitive exams. Therefore, it is advisable that you learn them by heart.
Given below are some questions based on the above-discussed formulas.)

Question 1: Find the average of first 25 odd numbers.
(1) 20
(2) 24
(3) 27
(4) 25

Answer and Explanation → first find the sum of 25 odd numbers (1+3+5+7+9+……+47+49) and the formula to calculate the sum is n^2 where n represent the number of terms which is 25 so sum = $25^2 = 625$

Average = $\frac{sum\ of\ the\ observation}{number\ of\ terms}$ =$\frac{625}{25}$ = 25

Question 2: Find the average of first 40 even numbers.
(1) 40
(2) 41
(3) 46
(4) 44

Answer and Explanation → first find the sum of 40 even numbers (2+4+6+8+10+……+78+80) and the formula to calculate the sum is n (n+1) where n represent the number of terms which is 40 so sum = 40 (40+1) = 1640
Average = $\frac{sum\ of\ the\ observation}{number\ of\ terms}$ =$\frac{1640}{40}$ = 41

Question 3: Find average of odd numbers from 1 to 200.
(1) 200
(2) 150
(3) 100
(4) 140

Answer and Explanation→ first find the sum of 100 odd numbers (1+3+5+7+9+……+197+199) and the formula to calculate the sum is n^2 where n represent the number of terms which is 100 so sum = 100^2= 10000
Average = $\frac{sum\ of\ the\ observation}{number\ of\ terms}$ =$\frac{10000}{100}$ = 100

Question 4: Find average of even numbers from 1 to 300.
(1) 151
(2) 161
(3) 171
(4)181

Answer and Explanation → first find the sum of 150 even numbers (2+4+6+8+10+……+298+300) and the formula to calculate the sum is n (n+1) where n represent the number of terms which is 150 so sum = 150 (150+1) = 22650
Average = $\frac{sum\ of\ the\ observation}{number\ of\ terms}$ =$\frac{122650}{150}$ =151

Concept 2

When a number is wrongly taken while solving average.
If average of n quantities $a_1, a_2, a_3 \ldots \ldots \ldots a_n$ is A_1. It was later discovered that while calculating average, one number 'a' was wrongly read as a_1, so the correct average will be:

Correct average = wrong average + $\frac{correct\ number - wrong\ number}{n}$

Derivation:

Let the wrong average = $A_1$and wrongly read number = a_1

Let The correct average = $A_2$and the correct number = a

$$\frac{a_1+a_2+a_3.....+a_n}{n} = A_1.....(1)$$

$$\frac{a+a_2+a_3.....+a_n}{n} = A_2.....(2)$$

Subtracting (2) from (1), we get:

$$a_1 + nA_2 - a = nA_1$$

$$A_2 = \frac{nA_1+a - a_1}{n}$$

$$A_2 = \frac{nA_1}{n}+\frac{a - a_1}{n}$$

$$A_2 = A_1+\frac{a - a_1}{n}$$

Correct average = wrong average + $\frac{correct\ number - wrong\ number}{n}$

Question 5: The average of 10 numbers is calculated as 15. It is later discovered that while calculating the average, one number namely 36, was wrongly read as 26. The correct average is?

(1) 20
(2) 18
(3) 16
(4) 14

Answer and Explanation → To find the answer in a second to apply this given formula

Correct average = wrong average + $\frac{correct\ number - wrong\ number}{n}$ → $15 + \frac{36-26}{10} =$ 16.