



ALGEBRA

Algebra is one of the oldest branches in the history of mathematics that **deals with number theory, geometry, and analysis**. The definition of algebra sometimes states that the study of the mathematical symbols and the rules involves manipulating these mathematical symbols. Algebra includes almost everything right from solving elementary equations to the study of abstractions. Algebra equations are included in many chapters of Maths, which students will learn in their academics. Also, there are several formulas and identities present in algebra.

What is Algebra?

Algebra helps solve the mathematical equations and allows to derive unknown quantities, like the bank interest, proportions, percentages. We can use the variables in the algebra to represent the unknown quantities that are coupled in such a way as to rewrite the equations.

The algebraic formulas are used in our daily lives to find the distance and volume of containers and figure out the sales prices as and when needed. Algebra is constructive in stating a mathematical equation and relationship by using letters or other symbols representing the entities. The unknown quantities in the equation can be solved through algebra.

Some of the main topics coming under algebra include Basics of algebra, exponents, simplification of algebraic expressions, polynomials, quadratic equations, etc.

ALGEBRA BASIC FORMULA

There are some important formulas are given, which'll help you to solving the algebraic problem. The algebra problems will involve expressions, polynomials, the system of equations, real numbers, inequalities, etc. Learn more algebra formulas that are used in Maths.

$$\begin{aligned}
 (a + b)^2 &= a^2 + b^2 + 2ab &= (a - b)^2 + 4ab \\
 (a - b)^2 &= a^2 + b^2 - 2ab &= (a - b)^2 - 4ab \\
 (a^2 - b^2) &= (a + b)(a - b) \\
 (a + b)^2 - (a - b)^2 &= 2(a^2 + b^2) \\
 (a + b)^2 - (a - b)^2 &= 4ab \\
 (a + b)^3 &= a^3 + b^3 + 3ab(a + b) \\
 (a + b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3 \\
 (a - b)^3 &= a^3 - b^3 - 3ab(a - b)
 \end{aligned}$$

$$\begin{aligned}
(a - b)^3 &= a^3 - 3a^2b - 3ab^2 - b^3 \\
(a^3 + b^3) &= (a + b)(a^2 + b^2 - ab) \\
(a^3 - b^3) &= (a - b)(a^2 + b^2 + ab) \\
(a^3 + b^3 + c^3 - 3abc) &= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \\
(a^3 + b^3 + c^3 - 3abc) &= \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2] \\
(a^3 + b^3 + c^3 - 3abc) &= \frac{1}{2}(a + b + c)[3(a^2 + b^2 + c^2 - (a + b + c)^2)] \\
(a^4 + a^2b^2 + b^4) &= (a^2 + ab + b^2)(a^2 - ab + b^2) \\
(a + b + c)^2 &= (a^2 + b^2 + c^2 + 2ab + 2bc + 2ca) \\
(a - b - c)^2 &= a^2 + b^2 + c^2 + 2(-ab + bc - ca) \\
(a - b - c)^2 &= a^2 + b^2 + c^2 - 2ab + 2bc - 2ca
\end{aligned}$$

Basic Changes

$$\begin{aligned}
(a + b)^3 + (a - b)^3 &= 2(a^3 + 3ab^2) \\
(a + b)^3 + (a - b)^3 &= 2(a^3 + 3a^2b) \\
(a^2 - b^2)^2 &= a^4 + b^4 - 2a^2b^2 \\
(a^2 - b^2)^2 - a^2b^2 &= a^4 + b^4 - 3a^2b^2 \\
(a^2 - b^2 - ab)(a^2 - b^2 + ab) &= a^4 + b^4 - 3a^2b^2 \\
(a^3 + b^3) &= (a + b)(a^2 + b^2 - ab) \\
(a^3 + b^3) &= (a + b)[(a + b)^2 - 3ab] \\
(a^3 + b^3) &= (a + b)[(a + b)^2 - (\sqrt{3ab})^2] \\
(a^3 + b^3) &= (a + b)(a + b) + \sqrt{3ab}(a + b - \sqrt{3ab}) \\
(a^4 + b^4) &= (a^2 + b^2)^2 - 2a^2b^2 \\
&= (a^2 + b^2)^2 - (\sqrt{2ab})^2 \\
&= (a^2 + b^2 + \sqrt{2ab})(a^2 + b^2 - \sqrt{2ab})
\end{aligned}$$

- If $a + b + c = 0$ is given in the question then,
Put $a = 1, b = 1, c = -2$
- If $ab + bc + ca = 0$ is given in the question then,
Put $a = 2, b = 2, c = -1$
- If $a + b + c = 2s$ is given in the question then,
put $a = 1, b = 1, c = 2, s = 2$
- If $a^2 + b^2 + c^2 = 0$ is given in the question then,
put $a^2 = 1, b^2 = 1, c^2 = 2$

• If $x + \frac{1}{x} = a$, then

$$\begin{aligned} (1) \quad x^2 + \frac{1}{x^2} &= a^2 - 2 \\ (2) \quad x^3 + \frac{1}{x^3} &= a^3 - 3a \\ (3) \quad x^4 + \frac{1}{x^4} &= a^4 - 4a^2 + 2 \\ (4) \quad x^5 + \frac{1}{x^5} &= a^5 - 5a^3 + 5a \\ (5) \quad x - \frac{1}{x} &= \sqrt{a^2 - 4} \end{aligned}$$

• If $x - \frac{1}{x} = p$, then

$$\begin{aligned} (1) \quad x^2 + \frac{1}{x^2} &= p^2 + 2 \\ (2) \quad x^3 + \frac{1}{x^3} &= p^3 + 3p \\ (3) \quad x^4 + \frac{1}{x^4} &= p^4 + 4p^2 + 2 \\ (4) \quad x^5 - \frac{1}{x^5} &= p^5 + 5p^3 + 5p \\ (5) \quad x + \frac{1}{x} &= \sqrt{p^2 + 4} \end{aligned}$$

